MATH2050a Mathematical Analysis I

Exercise 5 suggested Solution

11. Use the definition of limit to prove the following.

(a) $\lim_{x\to 3}\frac{2x+3}{4x-9}=3$, (b)$\lim_{x\to 6}\frac{x^2-3x}{x+3}=2$.

Solution:

(a) Let $f(x)=\frac{2x+3}{4x-9}$, then $|f(x)-3|=|\frac{30-10x}{4x-9}|$. For each $\epsilon>0$, choose $\delta(\epsilon)=min\{1,\frac{\epsilon}{10}\}$, then $\forall x\in(3-\delta(\epsilon),3+\delta(\epsilon))$, we have

$$|\frac{1}{4x-9}|\leq 1,\qquad |30-10x|<\epsilon$$

Hence, $|f(x)-3|\leq 1\times\epsilon$, we have $\lim_{x\to 3}\frac{2x+3}{4x-9}=3$.

(b) Let $f(x)=\frac{x^2-3x}{x+3}$, then

$$|f(x)-2|=|\frac{x^2-5x-6}{x+3}|=|\frac{x+1}{x+3}||x-6|=|1-\frac{2}{x+3}||x-6|.$$

For each $\epsilon>0$, choose $\delta(\epsilon)=min\{1,\frac{\epsilon}{2}\}$, then $\forall x\in(6-\delta(\epsilon),6+\delta(\epsilon))$, we have

$$|1-\frac{2}{x+3}|\leq 1+\frac{2}{|x+3|}<2,\qquad |x-6|<\frac{\epsilon}{2}$$

Hence, $|f(x)-2|\leq 2\times\frac{\epsilon}{2}$, we have $\lim_{x\to 6}\frac{x^2-3x}{x+3}=2$.

12. Show that the following limit does not exist.

(c) $\lim_{x\to 0}(x+sgn(x))\quad (x>0)$, (d)$\lim_{x\to 0}sin(\frac{1}{x^2})$.

Solution:

(c) Let $\{x_n\}$ be a sequence, $x_n=\frac{1}{n}$, $n\in N$; $\{y_n\}$ be a sequence, $y_n=\frac{-1}{n}$, $n\in N$. Then we have

$$lim(x_n+sgn(x_n))=1,\qquad lim(y_n+sgn(y_n))=-1$$

Notice that $lim x_n=lim y_n=0$, by thm 4.1.9, $\lim_{x\to 0}(x+sgn(x))$ doesn't exist.

(d) Similarly, let $x_n=\frac{1}{\sqrt{n\pi}}$, $n\in N$; $y_n=\frac{1}{\sqrt{2n\pi+\pi/2}}$, $n\in N$. Then we have

$$lim sin(\frac{1}{x_n^2})=0,\qquad lim sin(\frac{1}{y_n^2})=1$$

Notice that $lim x_n=lim y_n=0$, by thm 4.1.9, $\lim_{x\to 0}sin(\frac{1}{x^2})$ doesn't exist.